

**Probability distribution over some phenomenological models
in the matrix model compactified on a torus**

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Abstract

We study some phenomenological models in a matrix model corresponding to the IIB matrix model compactified on a six-dimensional torus with magnetic fluxes. Extending our previous works, we examine a wider class of models: a Pati-Salam-like model with a gauge group $U(4) \times U_L(2) \times U_R(2)$, and models where the gauge group $U(4)$ is broken down to $U_c(3) \times U(1)$ and/or the $U_R(2)$ is broken to $U(1)$ ². We find all the matrix configurations that yield matter content of all the phenomenological models whose gauge group is a subgroup of $U(8)$. We then estimate semiclassically a probability distribution for the appearance of the phenomenological models.

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1 Introduction

The standard model (SM) of particle physics agrees well with experiments. While phenomenological models beyond the SM will be explored in (near-)future experiments, some theoretical guides may be helpful. It may also be important to reconsider why the SM is so successful and what we should ask nature next.

On the other hand, the SM is unsatisfactory as a final theory, and the string theory is expected to be an ultimate theory including gravity. Phenomenological models inspired by the string theories have been studied extensively (see, for instance, ref. [1]). However, a serious problem in the string theory is that it has too many vacua.

A candidate for a nonperturbative formulation of string theory is matrix models (MM) [2, 3, 4]. Since the MM has a definite action and measure, we can, in principle, dynamically compare the string vacua, and calculate everything, such as spacetime dimensions, gauge groups, and matter contents. Indeed, spacetime structures have been analyzed intensively, and four-dimensionality seems to be preferred in the IIB matrix model [5, 6, 7]. Then, assuming that our spacetime is obtained, we will consider how the SM and some phenomenological models appear from the MM, and estimate probability distribution of their appearance. Such phenomenological studies in the MM may give us a guide for exploring phenomenological models beyond the SM.

An important ingredient of the SM is the chirality of fermions. We usually obtain a chiral spectrum on our spacetime by introducing nontrivial topologies, which then give chiral zero modes, in the extra dimensions: Euler characteristics of compactified manifolds, special boundary conditions at orbifold singularities, the intersection numbers of D-branes, etc., give nontrivial topologies. Also from the MM, chiral fermions and the SM matter content were obtained by considering toroidal compactifications with magnetic fluxes [8, 9] and intersecting D-branes [10]¹. The former is similar to the magnetized D-branes wrapping on a torus, which are T-dual to the latter.

In this paper, we will study phenomenological models in the MM compactified on a torus, extending our previous works [8, 9]. We here examine a wider class of phenomenological models than in ref. [9]. In order to embed the SM fermions with three generations into matrices in our formulation, the gauge group must be a subgroup of $U(8)$ or larger groups. We then find all the matrix configurations that yield all the phenomenological models whose gauge group is a subgroup of $U(8)$. Exhausting all the solutions is necessary for studying a

¹ Studies based on fuzzy spheres were given in [11, 12, 13]. MM's for orbifolds and orientifolds were studied in [14, 15].

semiclassical analysis.

We here note that even if a matter field is massless at the tree level in the MM, which might be interpreted as phenomena at the Planck scale, it would obtain a mass through quantum corrections at low energies. While masses of the gauge field and fermionic field can be protected by the gauge and the chiral symmetry, respectively, a mass of a scalar field is difficult to protect, which is well-known as the naturalness or the hierarchy problem. Thus, although the matrices provide scalar fields with the same representation under the gauge group as the Higgs field, it is difficult to keep them massless. Then, we will first find matrix configurations that provide the gauge and the fermionic fields, assuming that the electroweak symmetry breaking is caused by those Higgs candidates or some other mechanisms. We will next assume situations where the Higgs mass is protected by the supersymmetry, and find matrix configurations that provide the Higgsino fields as well.

We then study the dynamics of MM semiclassically, and estimate a probability distribution for the appearance of the phenomenological models². The fact that we could perform these analyses is an advantage of MM³. On the other hand, there remain some important issues in the formulations of MM, such as the relations between MM and string theory, interpretations of spacetime and matter in the matrices, and how to take large-N limits. We can expect reversely that these phenomenological studies may give some hints for those problems in MM.

In section 2, we briefly review a formulation of topological configurations on a torus, while a similar review was given in ref. [9]. We then find matrix configurations that provide the phenomenological models without the Higgsino fields in section 3, and those with the Higgsino fields in section 4. In section 5, we estimate the probability distribution over the phenomenological models. Section 6 is devoted to conclusions and discussions. In appendix A, detailed calculations for determining the fluxes are shown.

2 Review of topological configurations on a torus

We begin with a brief review of the IIB MM [3]. Its action has a simple form

$$S_{\text{IIBMM}} = -\frac{1}{g_{\text{IIBMM}}^2} \text{tr} \left(\frac{1}{4} [A_M, A_N] [A^M, A^N] + \frac{1}{2} \bar{\Psi} \Gamma^M [A_M, \Psi] \right), \quad (2.1)$$

² Probability distributions over the string landscape [16] were estimated, for instance, by counting the number of flux vacua [17], and by considering cosmological evolutions [18].

³ A related work is given in the MM of noncritical strings [19].

where A_M and Ψ are $N \times N$ Hermitian matrices. They are also a ten-dimensional vector and a Majorana-Weyl spinor, respectively. Performing a kind of functional integration

$$\int dA d\Psi e^{-S_{\text{IIBMM}}} \quad (2.2)$$

as a statistical system, and taking a suitable large- N limit, one can obtain a nonperturbative formulation of string theory. Since the measure as well as the action is defined definitely, we can calculate everything in principle.

A notable feature is that both spacetime and matter emerge from the matrices. Spacetime can be interpreted as an eigenvalue distribution of the bosonic matrices A_M , and its structures have been analyzed dynamically [5, 6, 7]. Arguments for the local fields on it were given [20]. It was also shown that non-commutative (NC) space and matter fields on it are described in MM rather elegantly [21, 22]⁴.

We now assume compactifications to $M^4 \times X^6$ with X^6 carrying nontrivial topologies. For concreteness, we consider toroidal compactifications of $M^4 \times T^6$. Toroidal compactifications were studied in Hermitian matrices [24, 21] and in unitary matrices [25]. We here use a finite-unitary-matrix formulation for NC tori. It can be defined by the twisted Eguchi-Kawai model [26, 27] (see, for instance, ref. [28]). Note, however, that such details of formulations are not relevant for studying phenomenological models. Any compactifications with nontrivial topologies can work as well. We then consider background configurations corresponding to

$$\begin{aligned} A_\mu &\sim x_\mu \otimes \mathbb{1} , \\ e^{iA_i} &\sim \mathbb{1} \otimes V_i , \end{aligned} \quad (2.3)$$

with $\mu = 0, \dots, 3$ and $i = 4, \dots, 9$. Matrices V_i represent T^6 , while x_μ represent our spacetime M^4 . We do not specify details of our spacetime x_μ , i.e., whether it is commutative or NC, compactified or not. A more precise correspondence between the IIB MM and the unitary MM will be given in section 5.

We now focus on V_i in (2.3), i.e., NC T^6 with nontrivial topologies. It is known that nontrivial topological sectors are defined by the so-called modules in NC geometries (see, for instance, ref. [29]). In the MM formulations, the modules are defined by imposing twisted boundary conditions on the matrices [28, 30]. In fact, each theory with twisted boundary conditions yields a single topological sector specified by the boundary conditions [31, 32], while in ordinary gauge theories on commutative spaces, a theory, for instance, with

⁴ It was also shown that curved spaces can be described by interpreting the matrices as differential operators [23].

periodic boundary conditions, provides all the topological sectors. However, since we now want to derive everything from the IIB MM, those topological features of NC gauge theories are not desirable. We thus introduce nontrivial topological sectors by background matrix configurations, not by imposing twisted boundary conditions by hand. Nontrivial topologies can be given by block-diagonal matrices [8]. We then consider the following configurations:

$$\begin{aligned}
V_{3+j} &= \begin{pmatrix} \Gamma_{1,j}^1 \otimes \mathbb{1}_{n_2^1} \otimes \mathbb{1}_{n_3^1} \otimes \mathbb{1}_{p^1} & & \\ & \ddots & \\ & & \Gamma_{1,j}^h \otimes \mathbb{1}_{n_2^h} \otimes \mathbb{1}_{n_3^h} \otimes \mathbb{1}_{p^h} \end{pmatrix}, \\
V_{5+j} &= \begin{pmatrix} \mathbb{1}_{n_1^1} \otimes \Gamma_{2,j}^1 \otimes \mathbb{1}_{n_3^1} \otimes \mathbb{1}_{p^1} & & \\ & \ddots & \\ & & \mathbb{1}_{n_1^h} \otimes \Gamma_{2,j}^h \otimes \mathbb{1}_{n_3^h} \otimes \mathbb{1}_{p^h} \end{pmatrix}, \\
V_{7+j} &= \begin{pmatrix} \mathbb{1}_{n_1^1} \otimes \mathbb{1}_{n_2^1} \otimes \Gamma_{3,j}^1 \otimes \mathbb{1}_{p^1} & & \\ & \ddots & \\ & & \mathbb{1}_{n_1^h} \otimes \mathbb{1}_{n_2^h} \otimes \Gamma_{3,j}^h \otimes \mathbb{1}_{p^h} \end{pmatrix},
\end{aligned} \tag{2.4}$$

with $j = 1, 2$. The number of blocks is denoted by h . Each block is a tensor product of four factors. The first three factors each represent T^2 of $T^6 = T^2 \times T^2 \times T^2$, and the last factor provides a gauge group structure. The configuration (2.4) gives the gauge group $U(p^1) \times U(p^2) \times \dots \times U(p^h)$.

The matrices $\Gamma_{l,j}^a$ with $a = 1, \dots, h$ and $l = 1, 2, 3$ in (2.4) are defined by using the Morita equivalence. For details, see, for instance, ref. [29, 28, 30, 8]. $\Gamma_{l,j}^a$ are $U(n_l^a)$ matrices that satisfy the 't Hooft-Weyl algebra

$$\Gamma_{l,1}^a \Gamma_{l,2}^a = e^{-2\pi i \frac{m_l^a}{n_l^a}} \Gamma_{l,2}^a \Gamma_{l,1}^a, \tag{2.5}$$

where the integers m_l^a and n_l^a are specified by an integer q_l^a as

$$m_l^a = -s_l + k_l q_l^a, \quad n_l^a = N_l - 2r_l q_l^a, \tag{2.6}$$

for each a and l . The integers N_l , r_l , s_l and k_l for each l specify the original torus of the Morita equivalence for each T^2 . Equations (2.6) can be inverted as⁵

$$1 = 2r_l m_l^a + k_l n_l^a, \quad q_l^a = N_l m_l^a + s_l n_l^a. \tag{2.7}$$

⁵ In the notations of p and q in ref. [29, 28, 30], the present case corresponds to $\tilde{p} = 1$, $\tilde{q} = q_l^a$, and thus $p_0 = p^a$, and thus $p = p^a$ and $q = p^a q_l^a$. Since the configurations (2.4) depend only on the dual tori, the original torus seems virtual. It is introduced just for treating all the dual tori equally. One could regard one of the dual tori as an original torus.

The fermionic matrix Ψ is similarly decomposed into blocks as

$$\Psi = \begin{pmatrix} \varphi^{11} \otimes \psi^{11} & \dots & \varphi^{1h} \otimes \psi^{1h} \\ \vdots & \ddots & \vdots \\ \varphi^{h1} \otimes \psi^{h1} & \dots & \varphi^{hh} \otimes \psi^{hh} \end{pmatrix}, \quad (2.8)$$

where φ^{ab} and ψ^{ab} represent spinor fields on M^4 and T^6 , respectively. Each block $\varphi^{ab} \otimes \psi^{ab}$ is in a bifundamental representation (p^a, \bar{p}^b) under the gauge group $U(p^a) \times U(p^b)$. It turns out [8] that ψ^{ab} has a topological charge on T^6 as

$$p^a p^b \prod_{l=1}^3 (n_l^b m_l^a - m_l^b n_l^a) = p^a p^b \prod_{l=1}^3 (q_l^a - q_l^b) = p^a p^b \prod_{l=1}^3 \left(-\frac{1}{2r} (n_l^a - n_l^b) \right). \quad (2.9)$$

Indeed, by defining an overlap-Dirac operator, which satisfies a Ginsparg-Wilson relation and an index theorem⁶, the Dirac index, i.e., the difference between the numbers of chiral zero modes, was shown to take the corresponding values⁷. In the present paper, we do not specify forms of the Dirac operator, and just assume that the correct number of chiral zero modes arises in the large- N limit or in the low energy effective theory.

3 Matrix configurations for some phenomenological models without the Higgsino fields

We now study matrix configurations that provide matter content of some phenomenological models.

As we saw in the previous section, in the present formulation, one can realize models with a gauge group $U(p^1) \times U(p^2) \times \dots \times U(p^h)$ and bifundamental matter fields. Unfortunately, as we showed in ref. [9], in the model with the gauge group $U(3) \times U(2) \times U(1)^2 \simeq SU(3) \times SU(2) \times U(1)^4 \subset U(7)$, either the right-handed singlet neutrino or the B-L gauge field cannot be included in the model. Neither do we have any solution for the fluxes that provide the SM fermions with three generations, in any models whose gauge group is a subgroup of $U(7)$ or smaller groups.

We then consider models whose gauge group is a subgroup of $U(8)$. All the subgroups of $U(8)$, which yield the SM fermions with three generations, are $U(4) \times U_L(2) \times U_R(2)$, $U_c(3) \times U(1) \times U_L(2) \times U_R(2)$, $U(4) \times U_L(2) \times U(1)^2$, and $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$. We will examine all of them. Since the

⁶ These techniques were developed in the lattice gauge theories [33] and applied to MM and NC geometries [34].

⁷ The same results were obtained in the fuzzy spheres [35, 13].

$U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model turns out to have no solution, we further study a model with $U_c(3) \times U_L(2) \times U(1)^4 \subset U(9)$, in order to find a model where the extra gauge group is Abelian.

As for the Higgs field, the bosonic matrices in the extra dimensions V_i , i.e., the gauge fields in the extra dimensions, give scalar fields on our spacetime. When the matrices are decomposed into the blocks, some of the block elements have the same representation under the gauge group as the Higgs field. However, the off-diagonal blocks give massive fields in general. A more serious problem is that, as we mentioned in the introduction, even if scalar fields are massless at the tree level, it is difficult to keep them massless against quantum corrections, which is well-known as the naturalness or the hierarchy problem. In the gauge-Higgs unifications [36], higher-dimensional gauge symmetries protect the scalar mass from the quadratic divergences of the cutoff order, but it still can receive quantum corrections of the order of the Kaluza-Klein scale (see also ref. [37]).

In this section, we will find matrix configurations that provide the gauge fields and the SM fermions, assuming that the above-mentioned Higgs candidates or some other mechanisms cause the electroweak symmetry breaking.

3.1 $U(4) \times U_L(2) \times U_R(2)$ model

We first consider the case with the number of blocks being three, i.e., $h = 3$, and the integers p^a taken to be 4, 2, 2 for $a = 1, \dots, h$. The gauge group is then $U(4) \times U(2) \times U(2)$.

The fermionic species are embedded in the fermionic matrix ψ as

$$\psi = \left(\begin{array}{c|cc|cc} & q & & u & d \\ o & l & & \nu & e \\ \hline & o & & o & \\ \hline & & & o & \end{array} \right), \quad (3.1)$$

where q denotes the quark doublets, l the lepton doublets, u and d the quark singlets, and ν and e the lepton singlets. The entries denoted as o give no massless fermions since, as we will see below, they are set to have a vanishing index. The lower triangle part can be obtained from the upper part by the charge conjugation transformation.

The fields q and l are in the $(4, \bar{2}, 1)$ representation under the gauge group $U(4) \times U_L(2) \times U_R(2)$. The fields u , d , ν , and e are in $(4, 1, \bar{2})$. As we will see in subsection 3.4, the fermionic fields have the correct representation under the SM gauge group $SU_c(3) \times SU_L(2) \times U_Y(1)$, which is a subgroup of $U(4) \times U_L(2) \times U_R(2)$. The color group $SU_c(3)$ and the lepton number $U(1)$ are unified to $SU(4)$, which is reminiscent of the Pati-Salam model [38].

We now determine the integers q_l^a specifying the magnetic fluxes. From (2.9), only the differences $q_l^a - q_l^b$ are relevant to the topology for the block ψ^{ab} . We thus define

$$q_l^{ab} = q_l^a - q_l^b, \quad (3.2)$$

$$q^{ab} = \prod_{l=1}^3 q_l^{ab}. \quad (3.3)$$

In order for (3.1) to have the correct generation number, the integers q^{ab} must have the values

$$q^{ab} = \begin{pmatrix} 0 & -3 & 3 \\ & 0 & 0 \\ & & 0 \end{pmatrix}. \quad (3.4)$$

The lower triangle part is obtained from the upper part by the relation $q^{ab} = -q^{ba}$. The block component with a vanishing index gives no chiral zero modes, and thus no massless fermions on our spacetime. Even when there are right-handed and left-handed zero modes in a pair, they will obtain a mass through quantum corrections, though they are massless at the tree level.

The fluxes in each T^2 , q_l^{ab} , are obtained by solving eq. (3.3) with the T^6 fluxes (3.4). Here, we note that eq. (3.3) is invariant under the permutations among q_1^{ab} , q_2^{ab} , and q_3^{ab} , and also under the sign flips: $q_1^{ab} \rightarrow -q_1^{ab}$, $q_2^{ab} \rightarrow -q_2^{ab}$, $q_3^{ab} \rightarrow q_3^{ab}$; $q_1^{ab} \rightarrow -q_1^{ab}$, $q_2^{ab} \rightarrow q_2^{ab}$, $q_3^{ab} \rightarrow -q_3^{ab}$; $q_1^{ab} \rightarrow q_1^{ab}$, $q_2^{ab} \rightarrow -q_2^{ab}$, $q_3^{ab} \rightarrow -q_3^{ab}$. By using these symmetries, we can fix the order of q_1^{ab} , q_2^{ab} , and q_3^{ab} , and their overall signs. Under this constraint, there are four solutions for q_l^{ab} . We list them in Table 1. In order to save space, we have omitted the diagonal elements that always vanish. We will call those matrices \hat{q}_l^{ab} .

3.2 $U_c(3) \times U(1) \times U_L(2) \times U_R(2)$ model

We then consider the case where the gauge group $U(4)$ in the previous subsection is broken down to $U_c(3) \times U(1)$ by the fluxes, i.e., the case with $h = 4$ and $p^a = (3, 1, 2, 2)$.

The fermionic species are embedded in the fermionic matrix ψ as

$$\psi = \begin{pmatrix} o & o & q & u & d \\ & o & l & \nu & e \\ & & o & o \\ & & & o \end{pmatrix}. \quad (3.5)$$

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}
$\begin{pmatrix} -1 & 1 \\ & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 \\ & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 \\ & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & -3 \\ & -6 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 \\ & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 \\ & -4 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 \\ & -2 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 \\ & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 \\ & 2 \end{pmatrix}$	$\begin{pmatrix} 3 & -1 \\ & -4 \end{pmatrix}$

Table 1: Fluxes in each T^2 in the $U(4) \times U_L(2) \times U_R(2)$ model, i.e., all the solutions for eq. (3.3) with the T^6 fluxes (3.4). The diagonal elements are omitted.

The fluxes in T^6 must have the values

$$q^{ab} = \begin{pmatrix} 0 & 0 & -3 & 3 \\ & 0 & -3 & 3 \\ & & 0 & 0 \\ & & & 0 \end{pmatrix}. \quad (3.6)$$

The fluxes in each T^2 , q_l^{ab} , are obtained by solving eq. (3.3) with the T^6 fluxes (3.6). They can be obtained by doubling the first row of the matrices in Table 1. However, if $q_l^{12} = 0$ for all l , which is equivalent to $q_l^1 = q_l^2$ for all l , the first diagonal block and the second one in the bosonic configuration (2.4) become identical, and the gauge group is enhanced from $U(3) \times U(1)$ to $U(4)$, which brings us back to the case in the previous subsection. Hence, we must find a solution that has both $q_l^{12} = 0$ and $q_l^{12} \neq 0$, depending on l . There are four solutions, which we list in Table 2.

3.3 $U(4) \times U_L(2) \times U(1)^2$ model

We next consider the case where the gauge group $U_R(2)$ is broken down to $U(1) \times U(1)$ by the fluxes, i.e., the case with $h = 4$ and $p^a = (4, 2, 1, 1)$.

The fermionic species are embedded in the fermionic matrix ψ as

$$\psi = \begin{pmatrix} o & q & u & d \\ & l & \nu & e \\ & o & o & o \\ & & o & o \\ & & & o \end{pmatrix}, \quad (3.7)$$

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}
$\begin{pmatrix} -2 & -1 & -1 \\ & 1 & 1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} -2 & -1 & -1 \\ & 1 & 1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -3 & 3 \\ & -3 & 3 \\ & & 6 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 & 1 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & 1 & 1 \\ & -1 & -1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 6 & 3 & 3 \\ & -3 & -3 \\ & & 0 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 & 1 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 4 & 3 & 3 \\ & -1 & -1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 4 & 1 & 1 \\ & -3 & -3 \\ & & 0 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 & 1 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & 3 & 3 \\ & 1 & 1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} -2 & 1 & 1 \\ & 3 & 3 \\ & & 0 \end{pmatrix}$

Table 2: Fluxes in each T^2 in the $U_c(3) \times U(1) \times U_L(2) \times U_R(2)$ model, i.e., all the solutions for eq. (3.3) with the T^6 fluxes (3.6). The diagonal elements are omitted.

The fluxes in T^6 must have the values

$$q^{ab} = \begin{pmatrix} 0 & -3 & 3 & 3 \\ & 0 & 0 & 0 \\ & & 0 & 0 \\ & & & 0 \end{pmatrix}. \quad (3.8)$$

The fluxes in each T^2 , q_t^{ab} , are obtained by solving eq. (3.3) with the T^6 fluxes (3.8). They can be obtained by suitably doubling the last column of the matrices in Table 1. Under the constraints mentioned in the previous subsections, there are nine solutions, where we have also fixed the exchange between the last two columns. We list them in Table 3.

3.4 $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model

We now consider the case where both $U(4) \rightarrow U_c(3) \times U(1)$ and $U_R(2) \rightarrow U(1)^2$ take place by the fluxes, i.e., the case with $h = 5$ and $p^a = (3, 1, 2, 1, 1)$.

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}
$\begin{pmatrix} -1 & 1 & -1 \\ & 2 & 0 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & -1 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & 3 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & 1 & -1 \\ & 2 & 0 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & -3 \\ & 0 & -6 \\ & & -6 \end{pmatrix}$
$\begin{pmatrix} -1 & 1 & 3 \\ & 2 & 4 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & 1 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$
$\begin{pmatrix} -1 & 1 & -3 \\ & 2 & -2 \\ & & -4 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & -1 \\ & 0 & -4 \\ & & -4 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 & 3 \\ & 0 & 4 \\ & & 4 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & -3 & 1 \\ & -6 & -2 \\ & & 4 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 & -3 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & -3 & -1 \\ & -6 & -4 \\ & & 2 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 & 3 \\ & 0 & 4 \\ & & 4 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 & 1 \\ & -4 & 0 \\ & & 4 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & 1 \\ & -2 & -2 \\ & & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 & -1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 & 3 \\ & -4 & 2 \\ & & 6 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & -1 \\ & -2 & -4 \\ & & -2 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 & -3 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 & 1 \\ & 2 & 0 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 3 & -1 & -1 \\ & -4 & -4 \\ & & 0 \end{pmatrix}$

Table 3: Fluxes in each T^2 in the $U(4) \times U_L(2) \times U(1) \times U(1)$ model, i.e., all the solutions for eq. (3.3) with the T^6 fluxes (3.8). The diagonal elements are omitted.

The fermionic species are embedded in the fermionic matrix ψ as

$$\psi = \left(\begin{array}{c|c|c|c|c} o & o & q & u & d \\ \hline & o & l & \nu & e \\ \hline & & o & o & o \\ \hline & & & o & o \\ \hline & & & & o \end{array} \right) . \quad (3.9)$$

The fluxes in T^6 must have the values

$$q^{ab} = \begin{pmatrix} 0 & 0 & -3 & 3 & 3 \\ & 0 & -3 & 3 & 3 \\ & & 0 & 0 & 0 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix} . \quad (3.10)$$

The fluxes in each T^2 , q_l^{ab} , are obtained by solving eq. (3.3) with the T^6 fluxes (3.10). They could be obtained by suitably doubling the last column of the matrices in Table 2, or by suitably doubling the first row of the matrices in Table 3. Unfortunately, there is no solution, however.

We then generalize the fermion embedding (3.9) to

$$\psi = \begin{pmatrix} o & u' & q & u & d \\ & o & l & \nu(\bar{\nu}) & e \\ & & o & \bar{l}' & o \\ & & & o & e' \\ & & & & o \end{pmatrix} , \quad (3.11)$$

$$\psi = \begin{pmatrix} o & u & q & d_1 & d_2 \\ & o & l & e_1 & e_2 \\ & & o & o & o \\ & & & o & \nu(\bar{\nu}) \\ & & & & o \end{pmatrix} , \quad (3.12)$$

though they cannot be applied to the models with the larger gauge groups in the previous subsections. Here, we have omitted the lines that separate the matrix elements since they are no longer necessary.

In fact, (3.11) and (3.12) are the most general embeddings where all the elements have the correct representation under the SM gauge group $SU_c(3) \times SU_L(2) \times U_Y(1)$: One can easily see that they have the correct representation under the $SU_c(3) \times SU_L(2)$. Since we have five $U(1)$ gauge groups coming from each diagonal block, we consider their linear combinations

$$\sum_{i=1}^5 x^i Q^i , \quad (3.13)$$

	q	u	u'	d	l	l'	$\nu(\bar{\nu})$	e	e'
Y	1/6	2/3	2/3	-1/3	-1/2	-1/2	0	-1	-1
B	1/3	1/3	1/3	1/3	0	0	0	0	0
L'	0	0	-1	0	1	0	1	1	0
Q_L	1	0	0	0	1	1	0	0	0
Q'_R	0	1	0	1	0	-1	1	1	0

	x^1	x^2	x^3	x^4	x^5
Y	1/6	-1/2	0	-1/2	1/2
B	1/3	0	0	0	0
L'	0	1	0	0	0
Q_L	0	0	-1	0	0
Q'_R	0	0	0	-1	-1

Table 4: U(1) charges for $q, u, u', d, l, l', \nu(\bar{\nu}), e$, and e' in (3.11), and the corresponding values for the coefficients x^i in (3.13).

where Q^i is the U(1) charge from the i th block. For the (3.11) case, we can consider the hypercharge Y , baryon number B , lepton number L' , left-handed charge Q_L , and right-handed charge Q'_R . Their charge for $q, u, u', d, l, l', \nu(\bar{\nu}), e$, and e' , and the corresponding values for x^i are given in Table 4. The charges L' and Q'_R have the proper interpretation when u', l' , and e' disappear, and ν , not $\bar{\nu}$, is chosen in Table 4, and thus in (3.11). This case reduces to the original embedding (3.9). This also ensures that the fermion species in (3.1), (3.5), and (3.7) in the previous subsections have the correct representation under the SM gauge group and the extra U(1) gauge groups, which are subgroups of the gauge group in the corresponding model.

For the (3.12) case, we consider the hypercharge Y , baryon number B , left-handed charge Q_L , and other two charges Q'_1 and Q'_2 . Their charge for $q, u, d_1, d_2, l, \nu(\bar{\nu}), e_1$, and e_2 , and the corresponding values for x^i are given in Table 5. The charges Q'_1 and Q'_2 have no proper interpretations. We also note that both in the cases (3.11) and (3.12), a linear combination of those five U(1) charges gives an overall U(1) and does not couple to the matter. Only four U(1) charges couple to the matter.

	q	u	d_1	d_2	l	$\nu(\bar{\nu})$	e_1	e_2
Y	1/6	2/3	-1/3	-1/3	-1/2	0	-1	-1
B	1/3	1/3	1/3	1/3	0	0	0	0
Q_L	1	0	0	0	1	0	0	0
Q'_1	0	0	1	0	0	0	1	0
Q'_2	0	0	0	1	0	1	0	1

	x^1	x^2	x^3	x^4	x^5
Y	1/6	-1/2	0	1/2	1/2
B	1/3	0	0	0	0
Q_L	0	0	-1	0	0
Q'_1	0	0	0	-1	0
Q'_2	0	0	0	0	-1

Table 5: U(1) charges for $q, u, d_1, d_2, l, \nu(\bar{\nu}), e_1$, and e_2 , in (3.12), and the corresponding values for the coefficients x^i in (3.13).

The fluxes in T^6 must have the values

$$q^{ab} = \begin{pmatrix} 0 & x & -3 & 3-x & 3 \\ & 0 & y-3 & \pm 3 & 3-z \\ & & 0 & y & 0 \\ & & & 0 & z \\ & & & & 0 \end{pmatrix} \quad (3.14)$$

for (3.11), and

$$q^{ab} = \begin{pmatrix} 0 & 3 & -3 & x & 3-x \\ & 0 & -3 & y & 3-y \\ & & 0 & 0 & 0 \\ & & & 0 & \pm 3 \\ & & & & 0 \end{pmatrix} \quad (3.15)$$

for (3.12). Here, x, y , and z can take an integer 0, 1, 2, or 3. Since the up-type quarks are embedded at the two places u and u' in (3.11), the corresponding fluxes can take several values x and $3-x$ in (3.14). The same is true for l and l' , and so on. The double sings of ± 3 are chosen whether ν or $\bar{\nu}$ is embedded in (3.11) and (3.12). The fluxes in each T^2 , q_l^{ab} , are obtained by solving eq. (3.3) with the T^6 fluxes (3.14) and (3.15). As we will show in Appendix A.2, there is no solution, either. We thus conclude that there is no solution in the $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model.

3.5 $U_c(3) \times U_L(2) \times U(1)^4$ model

We then consider the case with the gauge group $U_c(3) \times U_L(2) \times U(1)^4 \subset U(9)$, i.e., the case with $h = 6$ and $p^a = (3, 2, 1, 1, 1, 1)$, in order to find a model with the SM gauge group plus extra $U(1)$'s.

The fermionic species are embedded in the fermionic matrix ψ as

$$\psi = \begin{pmatrix} o & q & u & u & u & d \\ & o & \bar{l} & \bar{l} & \bar{l} & o \\ & & o & \nu & \nu & e \\ & & & o & \nu & e \\ & & & & o & e \\ & & & & & o \end{pmatrix}, \quad (3.16)$$

$$\psi = \begin{pmatrix} o & q & u & u & d & d \\ & o & \bar{l} & \bar{l} & o & o \\ & & o & \nu & e & e \\ & & & o & e & e \\ & & & & o & \nu \\ & & & & & o \end{pmatrix}, \quad (3.17)$$

$$\psi = \begin{pmatrix} o & q & u & d & d & d \\ & o & \bar{l} & o & o & o \\ & & o & e & e & e \\ & & & o & \nu & \nu \\ & & & & o & \nu \\ & & & & & o \end{pmatrix}, \quad (3.18)$$

where ν can be either ν or $\bar{\nu}$. They exhaust all the embeddings that have the correct representation under the SM gauge group.

The fluxes in T^6 , q^{ab} , can take several values for (3.16), (3.17), and (3.18), as in (3.14) and (3.15). We can also determine the fluxes for T^2 , q_l^{ab} , by solving eq. (3.3). As we will show in Appendix A.3, there are fifteen solutions for the case (3.16). The T^2 fluxes are listed in Table 16, Table 17, and Table 18. The corresponding T^6 fluxes are written in (A.15) to (A.20).

For the case (3.17), there are two solutions. The T^2 fluxes are listed in Table 15. The corresponding T^6 fluxes are (A.10) and (A.11). For the case (3.18), there is no solution.

4 Matrix configurations for the phenomenological models with the Higgsino fields

In this section, we assume situations where the Higgs mass is protected by the supersymmetry possessed by the IIB MM somehow. Then, we will find matrix configurations that yield candidates for the Higgsino fields as well as the gauge fields and the SM fermions. We just find candidates for fermion species, and do not study how the supersymmetry is realized in the whole spectrum.

4.1 $U(4) \times U_L(2) \times U_R(2)$ model

We first consider the model studied in subsection 3.1. The Higgsino fields h_u and h_d can be added to the fermion embedding (3.1) as⁸

$$\psi = \left(\begin{array}{c|cc|cc} o & q & u & d \\ & l & \nu & e \\ \hline & o & h_u & h_d \\ \hline & & & o \end{array} \right). \quad (4.1)$$

The T^6 fluxes must be altered from (3.4) to

$$q^{ab} = \begin{pmatrix} 0 & -3 & 3 \\ & 0 & q^H \\ & & 0 \end{pmatrix}, \quad (4.2)$$

with $q^H \neq 0$. There are three solutions for the T^2 fluxes. They are listed in Table 6, where we also write the values of q^H . Unfortunately, there is no solution with $q^H = 1$.

4.2 $U_c(3) \times U(1) \times U_L(2) \times U_R(2)$ model

We then consider the model studied in subsection 3.2. The Higgsino fields h_u and h_d are added to the embedding (3.5) as

$$\psi = \left(\begin{array}{c|cc|cc} o & o & q & u & d \\ \hline & o & l & \nu & e \\ \hline & & o & h_u & h_d \\ \hline & & & & o \end{array} \right). \quad (4.3)$$

⁸ A similar embedding was given in ref. [39].

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}	q^H
$\begin{pmatrix} -1 & 1 \\ & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 \\ & -2 \end{pmatrix}$	$\begin{pmatrix} 3 & -3 \\ & -6 \end{pmatrix}$	24
$\begin{pmatrix} -1 & 1 \\ & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 \\ & 2 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 \\ & -2 \end{pmatrix}$	-8
$\begin{pmatrix} -1 & 1 \\ & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 \\ & -4 \end{pmatrix}$	$\begin{pmatrix} 3 & -1 \\ & -4 \end{pmatrix}$	32

Table 6: Fluxes in each T^2 in the $U(4) \times U_L(2) \times U_R(2)$ model with the Higgsino candidates, i.e., all the solutions for the T^6 fluxes (4.2). The diagonal elements are omitted. The T^6 flux for the Higgsinos, q^H , is also listed.

The fluxes in T^6 have the values

$$q^{ab} = \begin{pmatrix} 0 & 0 & -3 & 3 \\ & 0 & -3 & 3 \\ & & 0 & q^H \\ & & & 0 \end{pmatrix}, \quad (4.4)$$

with $q^H \neq 0$. Solutions for the T^2 fluxes can be obtained by suitably doubling the first row of the matrices in Table 6. There are four solutions. They are listed in Table 7, where we also write the values of q^H .

Because the lepton doublet l and the Higgsino \bar{h}_u in (4.3) have the same representation under the SM gauge group, they can substitute for each other. Then, the T^6 fluxes can have more general values

$$q^{ab} = \begin{pmatrix} 0 & 0 & -3 & 3 \\ & 0 & q^L & 3 \\ & & 0 & q^H \\ & & & 0 \end{pmatrix}. \quad (4.5)$$

Solutions for the T^2 fluxes can also be obtained by using the the matrices in Table 6. There are fourteen solutions, other than those in Table 7. We do not present them here, in order to save space.

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}	q^H
$\begin{pmatrix} 2 & 1 & 3 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} -2 & -1 & 1 \\ & 1 & 3 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 0 & 3 & 1 \\ & 3 & 1 \\ & & -2 \end{pmatrix}$	-8
$\begin{pmatrix} 0 & -1 & 1 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} -4 & -3 & -1 \\ & 1 & 3 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} -4 & -1 & -3 \\ & 3 & 1 \\ & & -2 \end{pmatrix}$	-8
$\begin{pmatrix} -2 & -3 & -1 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 3 \\ & 1 & 3 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} -2 & 1 & -1 \\ & 3 & 1 \\ & & -2 \end{pmatrix}$	-8
$\begin{pmatrix} 0 & -1 & 1 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & 3 & -1 \\ & 1 & -3 \\ & & -4 \end{pmatrix}$	$\begin{pmatrix} -2 & 1 & -3 \\ & 3 & -1 \\ & & -4 \end{pmatrix}$	32

Table 7: Fluxes in each T^2 in the $U_c(3) \times U(1) \times U_L(2) \times U_R(2)$ model with the Higgsino candidates, i.e., all the solutions for the T^6 fluxes (4.4). The diagonal elements are omitted. The T^6 flux for the Higgsinos, q^H , is also listed.

4.3 $U(4) \times U_L(2) \times U(1)^2$ model

We next consider the model studied in subsection 3.3. The Higgsino fields h_u and h_d are added to the embedding (3.7) as

$$\left(\begin{array}{c|c|c|c} o & q & u & d \\ & l & \nu & e \\ \hline & o & h_u & h_d \\ \hline & & o & o \\ \hline & & & o \end{array} \right). \quad (4.6)$$

The fluxes in T^6 must have the values

$$q^{ab} = \begin{pmatrix} 0 & -3 & 3 & 3 \\ & 0 & q^{H_u} & q^{H_d} \\ & & 0 & 0 \\ & & & 0 \end{pmatrix}, \quad (4.7)$$

with $q^{H_u}, q^{H_d} \neq 0$. Solutions for the T^2 fluxes can be obtained by suitably doubling the last column of the matrices in Table 6. There are five solutions. They are listed in Table 8, where we also write the values of q^{H_u} and q^{H_d} . Solutions with $q^{H_u} = 0$ and $q^{H_d} \neq 0$, or $q^{H_u} \neq 0$ and $q^{H_d} = 0$, can also be obtained by suitably combining the last column of the matrices in Table 1 and

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}	q^{H_u}, q^{H_d}
$\begin{pmatrix} -1 & 1 & 1 \\ & 2 & 2 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & 3 \\ & -2 & 2 \\ & & 4 \end{pmatrix}$	$\begin{pmatrix} 3 & -3 & 1 \\ & -6 & -2 \\ & & 4 \end{pmatrix}$	24, -8
$\begin{pmatrix} -1 & 1 & 1 \\ & 2 & 2 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 & -3 \\ & 2 & -4 \\ & & -6 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & -1 \\ & -2 & -4 \\ & & -2 \end{pmatrix}$	-8, 32
$\begin{pmatrix} -1 & 1 & 1 \\ & 2 & 2 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 & -1 \\ & -4 & -2 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 3 & -1 & -3 \\ & -4 & -6 \\ & & -2 \end{pmatrix}$	32, 24
$\begin{pmatrix} -1 & 1 & -3 \\ & 2 & -2 \\ & & -4 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 & -1 \\ & 2 & -2 \\ & & -4 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & 1 \\ & -2 & -2 \\ & & 0 \end{pmatrix}$	-8, -8
$\begin{pmatrix} -1 & 1 & 3 \\ & 2 & 4 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 & -1 \\ & -4 & -2 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 3 & -1 & -1 \\ & -4 & -4 \\ & & 0 \end{pmatrix}$	32, 32

Table 8: Fluxes in each T^2 in the $U(4) \times U_L(2) \times U(1)^2$ model with the Higgsino candidates, i.e., solutions for the T^6 fluxes (4.7) (Part 1). The diagonal elements are omitted. Also listed are the T^6 fluxes for the Higgsinos, q^{H_u} and q^{H_d} , both of which take nonzero values.

Table 6. There are eight solutions. They are listed in Table 9, where we also write the values of q^{H_u} and q^{H_d} .

4.4 $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model

We now consider the model studied in subsection 3.4. The Higgsino fields h_u and h_d are added to the embedding (3.9) as

$$\psi = \left(\begin{array}{c|c|c|c|c} o & o & q & u & d \\ \hline & o & l & \nu & e \\ \hline & & o & h_u & h_d \\ \hline & & & o & o \\ \hline & & & & o \end{array} \right). \quad (4.8)$$

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}	q^{H_u}, q^{H_d}
$\begin{pmatrix} -1 & 1 & 1 \\ & 2 & 2 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & -1 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & -3 \\ & 0 & -6 \\ & & -6 \end{pmatrix}$	0, 24
$\begin{pmatrix} -1 & 1 & 1 \\ & 2 & 2 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 3 \\ & 0 & 2 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & 1 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$	0, -8
$\begin{pmatrix} -1 & 1 & 1 \\ & 2 & 2 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & -3 \\ & 0 & -4 \\ & & -4 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & -1 \\ & 0 & -4 \\ & & -4 \end{pmatrix}$	0, 32
$\begin{pmatrix} -1 & -1 & 1 \\ & 0 & 2 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & -1 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 3 & -3 & -3 \\ & -6 & -6 \\ & & 0 \end{pmatrix}$	0, 24
$\begin{pmatrix} -1 & -1 & 1 \\ & 0 & 2 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 & 3 \\ & -4 & 2 \\ & & 6 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & 1 \\ & -2 & -2 \\ & & 0 \end{pmatrix}$	0, -8
$\begin{pmatrix} -1 & -1 & 1 \\ & 0 & 2 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 & -3 \\ & -4 & -4 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & -1 \\ & -2 & -4 \\ & & -2 \end{pmatrix}$	0, 32
$\begin{pmatrix} -1 & -1 & 1 \\ & 0 & 2 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 & 3 \\ & 2 & 2 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & -1 & 1 \\ & -4 & -2 \\ & & 2 \end{pmatrix}$	0, -8
$\begin{pmatrix} -1 & -1 & 1 \\ & 0 & 2 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 & -3 \\ & 2 & -4 \\ & & -6 \end{pmatrix}$	$\begin{pmatrix} 3 & -1 & -1 \\ & -4 & -4 \\ & & 0 \end{pmatrix}$	0, 32

Table 9: Fluxes in each T^2 in the $U(4) \times U_L(2) \times U(1)^2$ model with the Higgsino candidates, i.e., solutions for the T^6 fluxes (4.7) (Part 2). The diagonal elements are omitted. Also listed are the T^6 fluxes for the Higgsinos, q^{H_u} and q^{H_d} , one of which takes nonzero values.

The fluxes in T^6 have the values

$$q^{ab} = \begin{pmatrix} 0 & 0 & -3 & 3 & 3 \\ & 0 & -3 & 3 & 3 \\ & & 0 & q^{H_u} & q^{H_d} \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}, \quad (4.9)$$

with $q^{H_u}, q^{H_d} \neq 0$. Solutions for the T^2 fluxes can be obtained by suitably doubling the last column of the matrices in Table 7, or by suitably doubling the first row of the matrices in Table 8. There is one solution. It is listed in the first row of Table 10, where we also write the values of q^{H_u} and q^{H_d} . Solutions with $q^{H_u} = 0$ and $q^{H_d} \neq 0$, or $q^{H_u} \neq 0$ and $q^{H_d} = 0$, can also be obtained by suitably combining the last column of the matrices in Table 2 and Table 7, or by suitably doubling the first row of the matrices in Table 9. There are two solutions, which are listed in the second and the third rows of Table 10.

As in the $U_c(3) \times U(1) \times U_L(2) \times U_R(2)$ model, since l and \bar{h}_u have the same representation under the SM gauge group, the T^6 fluxes can have more general values

$$q^{ab} = \begin{pmatrix} 0 & 0 & -3 & 3 & 3 \\ & 0 & q^L & \pm 3 & 3 \\ & & 0 & q^{H_u} & q^{H_d} \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}. \quad (4.10)$$

Solutions for the T^2 fluxes can be obtained by using the matrices in Table 8 and Table 9. There are three solutions. We list them in Table 11, where we also write the values of q^{H_u} , q^{H_d} and q^L . As for the double sign of ± 3 in (4.10), all of the solutions take $+3$.

5 Probability distribution over the phenomenological models

We now study the dynamics of MM semiclassically, and estimate the probabilities for the appearance of the phenomenological models obtained in the previous sections.

5.1 Model and instanton actions

We first specify the action of the MM. We here consider a ten-dimensional NC torus with an anisotropy of sizes between four and six dimensions, while

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}	q^{H_u}, q^{H_d}
$\begin{pmatrix} 2 & 1 & 3 & -1 \\ & -1 & 1 & -3 \\ & & 2 & -2 \\ & & & -4 \end{pmatrix}$	$\begin{pmatrix} -2 & -1 & 1 & -3 \\ & 1 & 3 & -1 \\ & & 2 & -2 \\ & & & -4 \end{pmatrix}$	$\begin{pmatrix} 0 & 3 & 1 & 1 \\ & 3 & 1 & 1 \\ & & -2 & -2 \\ & & & 0 \end{pmatrix}$	-8, -8
$\begin{pmatrix} 0 & -1 & 1 & 1 \\ & -1 & 1 & 1 \\ & & 2 & 2 \\ & & & 0 \end{pmatrix}$	$\begin{pmatrix} 4 & 3 & 3 & 1 \\ & -1 & -1 & -3 \\ & & 0 & -2 \\ & & & -2 \end{pmatrix}$	$\begin{pmatrix} 4 & 1 & 1 & 3 \\ & -3 & -3 & -1 \\ & & 0 & 2 \\ & & & 2 \end{pmatrix}$	0, -8
$\begin{pmatrix} 0 & -1 & 1 & 1 \\ & -1 & 1 & 1 \\ & & 2 & 2 \\ & & & 0 \end{pmatrix}$	$\begin{pmatrix} 2 & 3 & 3 & -1 \\ & 1 & 1 & -3 \\ & & 0 & -4 \\ & & & -4 \end{pmatrix}$	$\begin{pmatrix} -2 & 1 & 1 & -3 \\ & 3 & 3 & -1 \\ & & 0 & -4 \\ & & & -4 \end{pmatrix}$	0, 32

Table 10: Fluxes in each T^2 in the $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model with the Higgsino candidates. All the solutions for the T^6 fluxes (4.9). The diagonal elements are omitted. The T^6 fluxes for the Higgsinos, q^{H_u} and q^{H_d} , are also listed.

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}	q^{H_u}, q^{H_d}	q^L
$\begin{pmatrix} 0 & -1 & 1 & 1 \\ & -1 & 1 & 1 \\ & & 2 & 2 \\ & & & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & -1 & 3 \\ & 1 & -3 & 1 \\ & & -2 & 2 \\ & & & 4 \end{pmatrix}$	$\begin{pmatrix} -2 & 3 & -3 & 1 \\ & 5 & -1 & 3 \\ & & -6 & -2 \\ & & & 4 \end{pmatrix}$	24, -8	-5
$\begin{pmatrix} 0 & -1 & 1 & 1 \\ & -1 & 1 & 1 \\ & & 2 & 2 \\ & & & 0 \end{pmatrix}$	$\begin{pmatrix} -4 & 1 & -3 & -1 \\ & 5 & 1 & 3 \\ & & -4 & -2 \\ & & & 2 \end{pmatrix}$	$\begin{pmatrix} -4 & 3 & -1 & -3 \\ & 7 & 3 & 1 \\ & & -4 & -6 \\ & & & -2 \end{pmatrix}$	32, 24	-35
$\begin{pmatrix} 4 & -1 & 1 & 3 \\ & -5 & -3 & -1 \\ & & 2 & 4 \\ & & & 2 \end{pmatrix}$	$\begin{pmatrix} -4 & 1 & -3 & -1 \\ & 5 & 1 & 3 \\ & & -4 & -2 \\ & & & 2 \end{pmatrix}$	$\begin{pmatrix} 0 & 3 & -1 & -1 \\ & 3 & -1 & -1 \\ & & -4 & -4 \\ & & & 0 \end{pmatrix}$	32, 32	-75

Table 11: Fluxes in each T^2 in the $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model with the Higgsino candidates. All the solutions for the T^6 fluxes (4.10), other than those listed in Table 10. The diagonal elements are omitted. The T^6 fluxes for the Higgsinos and the lepton doublet, q^{H_u} , q^{H_d} and q^L , are also listed.

one can alternatively consider a six-dimensional torus with a four-dimensional uncompactified spacetime. The bosonic action is given by the twisted Eguchi-Kawai model [26, 27] (see, for instance, ref. [28]) as

$$S_b = -\beta \mathcal{N} \sum_{i \neq j} \mathcal{Z}_{ji} \text{tr} \left(\mathcal{V}_i \mathcal{V}_j \mathcal{V}_i^\dagger \mathcal{V}_j^\dagger \right) - \beta' \mathcal{N} \sum_{\mu \neq \nu} \mathcal{Z}_{\nu\mu} \text{tr} \left(\mathcal{V}_\mu \mathcal{V}_\nu \mathcal{V}_\mu^\dagger \mathcal{V}_\nu^\dagger \right) \\ - \beta'' \mathcal{N} \sum_{i\mu} \left[\mathcal{Z}_{\mu i} \text{tr} \left(\mathcal{V}_i \mathcal{V}_\mu \mathcal{V}_i^\dagger \mathcal{V}_\mu^\dagger \right) + \mathcal{Z}_{i\mu} \text{tr} \left(\mathcal{V}_\mu \mathcal{V}_i \mathcal{V}_\mu^\dagger \mathcal{V}_i^\dagger \right) \right], \quad (5.1)$$

where \mathcal{V}_μ and \mathcal{V}_i are $U(\mathcal{N})$ matrices with $\mu, \nu = 0, \dots, 3$ and $i, j = 4, \dots, 9$. We take twists

$$\mathcal{Z}_{01} = \exp \left(2\pi i \frac{s_4}{N_4} \right), \quad \mathcal{Z}_{23} = \exp \left(2\pi i \frac{s_5}{N_5} \right), \\ \mathcal{Z}_{45} = \exp \left(2\pi i \frac{s_1}{N_1} \right), \quad \mathcal{Z}_{67} = \exp \left(2\pi i \frac{s_2}{N_2} \right), \quad \mathcal{Z}_{89} = \exp \left(2\pi i \frac{s_3}{N_3} \right), \quad (5.2)$$

with a huge difference in N_l between $l = 4, 5$ and $l = 1, 2, 3$:

$$N_4 \simeq N_5 \gg N_1 \simeq N_2 \simeq N_3. \quad (5.3)$$

The other twists are taken to be one. The total matrix size \mathcal{N} is taken to be

$$\mathcal{N} = k \prod_{l=1}^5 N_l. \quad (5.4)$$

Note that the matrix size (5.4) is k times larger than is usually expected from the integers that specify the twists (5.2).

We now consider background configurations as

$$\mathcal{V}_\mu = V_\mu \otimes \mathbb{1}, \\ \mathcal{V}_i = \mathbb{1} \otimes V_i, \quad (5.5)$$

where V_μ are $U(N_4 N_5)$ matrices and V_i are $U(k N_1 N_2 N_3)$ matrices. The size of our spacetime and that of the extra dimensions are given by ϵN_l with $l = 4, 5$ and $l = 1, 2, 3$, respectively, where ϵ is a lattice spacing. They thus have a huge anisotropy due to (5.3).

We then consider the topological configurations (2.4) for V_i . In fact, they are classical solutions for the action (5.1) (see, for instance, ref. [40]). In order to match the matrix size,

$$\sum_{a=1}^h n_1^a n_2^a n_3^a p^a = k \prod_{l=1}^3 N_l \quad (5.6)$$

is required. Plugging (2.4) into (5.1), we obtain the classical action

$$S_b = -2\beta \mathcal{N} N_4 N_5 \sum_{l=1}^3 \sum_{a=1}^h n_1^a n_2^a n_3^a p^a \cos \left(2\pi \left(\frac{s_l}{N_l} + \frac{m_l^a}{n_l^a} \right) \right), \quad (5.7)$$

where we have written only the first term in (5.1).

Recall that the integers n_l^a and m_l^a are specified by an original torus with the integers N_l and s_l via (2.6) or (2.7). We now consider the configurations whose integers N_l and s_l agree with the ones that specify the twists \mathcal{Z}_{ij} in the action (5.1) via (5.2). The other configurations have much larger actions. Then, from (2.6) and (2.7), we can find the relation

$$\frac{s_l}{N_l} + \frac{m_l^a}{n_l^a} = \frac{q_l^a}{N_l n_l^a} = -\frac{1}{2r} \left(\frac{1}{N_l} - \frac{1}{n_l^a} \right) . \quad (5.8)$$

By plugging (5.8) into (5.7), we find that the classical action (5.7) takes the minimum value if and only if

$$q_l^a = 0 \Leftrightarrow n_l^a = N_l \quad (5.9)$$

for $\forall a$ and $\forall l$. Then, the constraint (5.6) becomes

$$\sum_{a=1}^h p^a = k . \quad (5.10)$$

Therefore, if we choose the parameters of the MM action (5.1) as in (5.4), block diagonal configurations, where the total number of the blocks is specified by (5.10), are dynamically favored.

We then consider small fluctuations around the minimum, i.e., configurations with $|q_l^a| \ll N_l$. The classical action (5.7) is approximated as

$$\Delta S_b \simeq 4\pi^2 \beta \frac{\mathcal{N}^2}{k} \sum_{l=1}^3 \frac{1}{(N_l)^4} \sum_{a=1}^h p^a (q_l^a)^2 , \quad (5.11)$$

where we have written the difference from the minimum value. We hereafter will call it an instanton action since it is a classical action of a topological configuration.

For comparison, let us consider cases with large fluctuations, for instance, configurations where the total number of blocks is different from (5.10), and configurations specified by original tori with the integers N_l , s_l which are different from the twists in the action (5.1). In those cases, the classical action (5.7) receives an enhancement factor of order $(N_l)^2$, compared to (5.11).

5.2 How to take large- N limits and the probability distribution over the string vacuum space

We now consider relations between the prescription of how to take a large- N limit in the MM and the probability distribution over various matrix configurations, i.e., various string vacua, based on the semiclassical analysis. For details, see ref. [9].

We first compare the IIB MM action (2.1) and the unitary MM action (5.1), by considering a correspondence between the Hermitian matrices and the unitary matrices as

$$\mathcal{V}_M \sim \exp \left(2\pi i \frac{A_M}{\epsilon N_l} \right). \quad (5.12)$$

By plugging (5.12) into (5.1), and comparing it with (2.1), we find a relation among the coupling constants in (5.1) and (2.1) as

$$\frac{1}{2} \beta \mathcal{N} \left(\frac{2\pi}{\epsilon N_l} \right)^4 = \frac{1}{g_{\text{IIBMM}}^2}, \quad (5.13)$$

with $l = 1, 2, 3$. Similar relations can be obtained for β' and β'' .

Since both (5.11) and (5.13) depend on $\beta/(N_l)^4$, by defining a combination

$$\frac{g_{\text{IIBMM}}^2}{\epsilon^4 \mathcal{N}} \equiv \frac{1}{A}, \quad (5.14)$$

the instanton action (5.11) becomes

$$\Delta S_b = \frac{A}{2\pi^2 k} \sum_{l=1}^3 \sum_{a=1}^h p^a (q_l^a)^2. \quad (5.15)$$

It then follows that scaling limits of fixing $g_{\text{IIBMM}}^2 \mathcal{N}^\alpha / \epsilon^4$ with $\alpha > -1$, $\alpha = -1$, and $\alpha < -1$ make the coefficient of the instanton action (5.15) infinite, finite, and vanishing values, respectively. Together with fixing the torus size $\epsilon \mathcal{N}^{1/5}$, those scaling limits correspond to fixing $g_{\text{IIBMM}}^2 \mathcal{N}^\gamma$ with $\gamma = \alpha + 4/5$. The three cases give drastically different results:

1. If we take a large- \mathcal{N} limit by fixing $g_{\text{IIBMM}}^2 \mathcal{N}^\alpha / \epsilon^4$ with $\alpha > -1$, or by fixing $g_{\text{IIBMM}}^2 \mathcal{N}^\gamma$ with $\gamma > -1/5$, the instanton action (5.15) diverges for nonzero q_l^a , and thus only a single topological sector survives. While in the present model, the topologically trivial sector, $q_l^a = 0$, is chosen, in more elaborated models, desirable sectors may be chosen uniquely by the dynamics. This is drastically different from the situations where physicists usually consider the landscape.
2. In a limit with $\alpha < -1$ or $\gamma < -1/5$, the instanton action (5.15) vanishes for all q_l^a , and all the topological sectors appear with equal probabilities. Then, the estimation for the probability distribution over the string vacuum space reduces to the number counting of the classical solutions. Moreover, in a limit with $\alpha < -1 - 2/5$, a still larger number of configurations, which were studied as the large fluctuations below (5.11), can also appear.

3. In a limit with $\alpha = -1$ or $\gamma = -1/5$, the instanton action (5.15) takes finite values for finite q_l^a , and all the topological sectors appear with finite and different probabilities.

5.3 Probabilities for the appearance of the phenomenological models

We now estimate the probabilities for the appearance of the phenomenological models obtained in sections 3 and 4.

We first consider the $U(4) \times U_L(2) \times U_R(2)$ model with the fluxes given by the first row in Table 1. By solving (3.2), q_l^a are determined as

$$\begin{aligned} q_1^a &= (q_1, q_1 + 1, q_1 - 1) , \\ q_2^a &= (q_2, q_2 - 1, q_2 - 1) , \\ q_3^a &= (q_3, q_3 - 3, q_3 - 3) , \end{aligned} \tag{5.16}$$

for $a = 1, \dots, h = 3$. Since only the differences are specified in (3.2), q_l^a are determined with arbitrary integer shifts q_1 , q_2 , and q_3 . The instanton action (5.15) takes various values depending on those arbitrary integers q_l . The minimum value

$$\Delta S_b = \frac{A}{2\pi^2 k} 28 , \tag{5.17}$$

is attained by $q_1 = 0$, $q_2 = 0$ or 1, and $q_3 = 1$ or 2.

One could further lower the instanton action (5.15) to

$$\Delta S_b = \frac{A}{2\pi^2 k} 24 , \tag{5.18}$$

by choosing the fractional values $q_1 = 0$, $q_2 = 1/2$, and $q_3 = 3/2$. This corresponds to modifying the twists in the action (5.1) from (5.2) to

$$\begin{aligned} \mathcal{Z}_{45} &= \exp \left(2\pi i \frac{s_1}{N_1} \right) , \\ \mathcal{Z}_{67} &= \exp \left(2\pi i \left(\frac{s_2}{N_2} + \frac{1}{2N_2^2} \right) \right) , \\ \mathcal{Z}_{89} &= \exp \left(2\pi i \left(\frac{s_3}{N_3} + \frac{1}{2N_3^2} \right) \right) , \end{aligned} \tag{5.19}$$

although they are not natural in the present MM.

The probability of the appearance of this model is semiclassically given as $e^{-\Delta S_b}$, multiplied by a factor coming from quantum corrections. There exist configurations with the same action, but with p^a and q_l^a different from (5.16), and thus the probability must be divided by this numerical factor.

In the same way, we estimate the minimum instanton action for the other cases in Table 1. We also examine the $U_c(3) \times U(1) \times U_L(2) \times U_R(2)$ model with the fluxes given in Table 2, and the $U(4) \times U_L(2) \times U(1)^2$ model with the fluxes in Table 3. The results are drawn in Table 12. We further examine the models with the Higgsino fields, studied in section 4, and list the results in Table 13. We do not present the results from the T^6 fluxes (4.5) and (4.10), since they give larger values of the instanton actions.

Within these models and in the integral q_l case, the minimum instanton action $\frac{2\pi^2 k}{A} \cdot \Delta S_b$ takes various values between 28 and 44 over the various models. The minimum of the minimum values, 28, is attained by the first solution in the $U(4) \times U_L(2) \times U_R(2)$ model, the last one in the $U_c(3) \times U(1) \times U_L(2) \times U_R(2)$ model, and the first one in the $U(4) \times U_L(2) \times U(1)^2$ model in Table 12, and also by the second solution in the $U(4) \times U_L(2) \times U_R(2)$ model and the seventh one in the $U(4) \times U_L(2) \times U(1)^2$ model in Table 13. We cannot find drastic differences among the models with various gauge groups, and between the models with and without the Higgsinos.

There also exist solutions with lower instanton actions, which do not yield the SM fermions. Moreover, as far as we consider the models whose gauge group is a subgroup of $U(8)$, i.e., the MM (5.1) with the parameter $k = 8$, the $U(8)$ model without any gauge symmetry breaking has the lowest action. In order to make phenomenologically-attractive models most probable, we need to elaborately modify the MM (5.1). However, we can also find that the phenomenological models have rather small instanton actions and are sufficiently probable if the coefficient A is not so large.

6 Conclusions and discussions

In this paper, we considered the situations where the IIB MM is compactified on a torus with magnetic fluxes, and exhausted all the matrix configurations that yield all the phenomenological models whose gauge group is a subgroup of $U(8)$, with and without the Higgsino fields. We then studied the dynamics of MM semiclassically, and estimated the probability distribution over the phenomenological models.

There remain some important problems. While we found the embedding of the Higgs field in the matrices, we need to consider how the electroweak symmetry breaking occurs. The gauge-Higgs unifications and the recombinations of the intersecting D-branes (see, for instance, ref. [41]) are close to the present situation and may be helpful to consider this issue. We should also study values of the Yukawa couplings and the flavor structure. They can be obtained by

gauge group	integral q_l	fractional q_l
U(8)	0	0
U(4) \times U _L (2) \times U _R (2)	28	24
	44	40
	36	32
	36	32
U _c (3) \times U(1) \times U _L (2) \times U _R (2)	44	43
	40	39
	36	36
	28	27
U(4) \times U _L (2) \times U(1) ²	28	25
	40	37
	32	29
	36	32
	44	40
	40	37
	36	36
	40	37
	36	33

Table 12: Values of the instanton actions $\frac{2\pi^2 k}{A} \cdot \Delta S_b$ in the U(8) model, the U(4) \times U_L(2) \times U_R(2) model, the U_c(3) \times U(1) \times U_L(2) \times U_R(2) model, and the U(4) \times U_L(2) \times U(1)² model, without the Higgsino fields. Each row of this table corresponds to those in Table 1, Table 2, and Table 3. The second column shows the minimum value within the integer values of q_l as in (5.17), while the third column is the results within the fractional values of q_l as in (5.18). Fractional values of q_l are not natural in the present MM.

gauge group	q^{H_u}, q^{H_d}	integral q_l	fractional q_l
$U(4) \times U_L(2) \times U_R(2)$	24	44	44
	-8	28	28
	32	44	40
$U_c(3) \times U(1) \times U_L(2) \times U_R(2)$	-8	32	31
	-8	44	40
	-8	32	31
	32	44	43
$U(4) \times U_L(2) \times U(1)^2$	24, -8	44	40
	-8, 32	40	39
	32, 24	44	43
	-8, -8	36	32
	32, 32	44	41
	0, 24	40	39
	0, -8	28	27
	0, 32	36	36
	0, 24	44	43
	0, -8	36	35
	0, 32	40	37
	0, -8	32	31
	0, 32	44	41
$U_c(3) \times U(1) \times U_L(2) \times U(1)^2$	-8, -8	36	35
	0, -8	40	39
	0, 32	40	39

Table 13: Values of the instanton actions $\frac{2\pi^2 k}{A} \cdot \Delta S_b$ in the the $U(4) \times U_L(2) \times U_R(2)$ model, the $U_c(3) \times U(1) \times U_L(2) \times U_R(2)$ model, the $U(4) \times U_L(2) \times U(1)^2$ model, and the $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model, with the Higgsino candidates. Each row of this table corresponds to those in Table 6, Table 7, Table 8, Table 9, and Table 10. We do not present the results from the T^6 fluxes (4.5) and (4.10), since they give larger values of the instanton actions. The T^6 fluxes for the Higgsinos, q^{H_u} and q^{H_d} , are also listed. The third column shows the minimum value within the integer values of q_l as in (5.17), while the fourth column is the results within the fractional values of q_l as in (5.18). Fractional values of q_l are not natural in the present MM.

calculating overlaps among the zero-mode fields. Related works were given, for instance, in ref. [42, 39].

While we considered situations where the supersymmetry protects the Higgs mass, we need to study how to keep a part of the supersymmetry possessed by the IIB MM and how to break it at low energies. It is also possible that the supersymmetry is broken at high energies but leave some remnants (see, for instance, ref. [43]). Other resolutions of the naturalness or the hierarchy problem are also interesting to consider, such as composite models, models with large-extra dimensions, some stringy or quantum-gravitational effects.

The models we considered in the present paper have extra $U(1)$ gauge groups and are anomalous within the gauge dynamics. This anomaly may be canceled via the Green-Schwarz mechanism by the exchange of RR-fields, which can also make the extra gauge fields massive. In order to realize this, some constraints as the RR tadpole cancelation condition should be imposed on MM. We should also generalize topological configurations by introducing the complex structures and the Wilson lines.

While we assumed toroidal compactifications and worked in a unitary matrix formulation in this paper, we should study relation between the unitary MM and the IIB MM, and how the parameters in the unitary MM action are determined from the IIB MM. We also need to consider how to interpret spacetime and matter in the matrices and how to describe compactifications in the matrices.

We will come back to these issues in future publications. We hope that these studies will give us some helps for both exploring the phenomenological models and studying the formulations of MM.

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A Solutions of the T^2 fluxes q_l^{ab}

In this appendix, we find solutions of the T^2 fluxes q_l^{ab} by solving eq. (3.3) in the $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model and the $U_c(3) \times U_L(2) \times U(1)^4$ model without the Higgsino fields, studied in section 3.

A.1 Two theorems

Before we solve eq. (3.3) in the concrete models, we prove two theorems.

Theorem 1: Pick up three elements from the T^6 fluxes and focus on q^{ab} , q^{bc} , and q^{ac} :

$$\begin{pmatrix} q^{ab} & q^{ac} \\ & q^{bc} \end{pmatrix}. \quad (\text{A.1})$$

Then, it is impossible that all of $|q^{ab}|$, $|q^{bc}|$, and $|q^{ac}|$ take 1, 2, or 3. In other words, if $|q^{ab}|$, $|q^{bc}|$, and $|q^{ac}|$ are 0, 1, 2, or 3, then, at least one of them must be 0. There is one exception: $(|q^{ab}|, |q^{bc}|, |q^{ac}|) = (2, 2, 2)$ is possible.

Proof: Let us first consider the case $(|q^{ab}|, |q^{bc}|) = (1, 1)$. Then, the T^2 fluxes $|q_l^{ab}|$ must take 1 for all l . So must $|q_l^{bc}|$. It then follows from $q_l^{ac} = q_l^{ab} + q_l^{bc}$ that $|q_l^{ac}|$ must be 0 or 2. Then, $|q^{ac}|$ is 0 or 8. Hence, $(|q^{ab}|, |q^{bc}|, |q^{ac}|) = (1, 1, 1)$, $(1, 1, 2)$, or $(1, 1, 3)$ is not possible.

Similarly, by considering the cases $(|q^{ab}|, |q^{bc}|) = (1, 2)$, $(1, 3)$, $(2, 2)$, $(2, 3)$, and $(3, 3)$, one can prove the theorem.

Theorem 2: Consider the T^6 fluxes

$$q^{ab} = \begin{pmatrix} 0 & 0 & -3 & 3 \\ & 0 & -3 & x \\ & & 0 & 0 \\ & & & 0 \end{pmatrix}, \quad (\text{A.2})$$

which is equivalent to

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 3 \\ & 0 & 3 & 0 \\ & & 0 & x \\ & & & 0 \end{pmatrix}, \quad (\text{A.3})$$

as can be seen by exchanging the second and third rows and columns. Then, it is impossible that x takes 0, 1, ± 2 , or -3 . Within $|x| \leq 3$, only $x = -1$ and $x = 3$ are possible.

Proof: For

$$q^{ab} = \begin{pmatrix} 0 & -3 & 3 \\ & 0 & 0 \\ & & 0 \end{pmatrix}, \quad (\text{A.4})$$

the solutions for eq. (3.3) are listed in Table 1. One can also list the solutions for

$$q^{ab} = \begin{pmatrix} 0 & -3 & x \\ & 0 & 0 \\ & & 0 \end{pmatrix}. \quad (\text{A.5})$$

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}
$\begin{pmatrix} -2 & -1 & -1 \\ & 1 & 1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 2 & 1 & 3 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 0 & 3 & -1 \\ & 3 & -1 \\ & & -4 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 & -1 \\ & -1 & -1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} -2 & 1 & -3 \\ & 3 & -1 \\ & & -4 \end{pmatrix}$	$\begin{pmatrix} 2 & 3 & 1 \\ & 1 & -1 \\ & & -2 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 & -1 \\ & -1 & -1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} -2 & 1 & -1 \\ & 3 & 1 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 2 & 3 & 3 \\ & 1 & 1 \\ & & 0 \end{pmatrix}$

Table 14: Fluxes in each T^2 for the T^6 fluxes (A.2) with $x = -1$. The diagonal elements are omitted.

By combining two solutions from each, one can construct a solution for (A.2). There is no solution for the cases $x = 0, 1, \pm 2, -3$. For $x = 3$, the solutions are listed in Table 2. For $x = -1$, there are three solutions, which we list in Table 14.

A.2 Fluxes in the $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model

We now study the $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model considered in subsection 3.4. One can immediately find that (3.15) has no solution for (3.3), by applying the Theorem 1 to $q^{12} = 3$, $q^{23} = -3$, and $q^{13} = -3$.

For (3.14), by applying the Theorem 1 to $q^{15} = 3$, one can find that $x = z = 0$ or $x = z = 3$ is allowed. Also, by applying the Theorem 1 to $q^{13} = -3$, $x = y = 0$ or $x = y = 3$ is allowed. Combining these two results, $x = y = z = 0$ or $x = y = z = 3$ is allowed. They correspond to

$$q^{ab} = \begin{pmatrix} 0 & 0 & -3 & 3 & 3 \\ & 0 & -3 & \pm 3 & 3 \\ & & 0 & 0 & 0 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}, \quad (\text{A.6})$$

$$q^{ab} = \begin{pmatrix} 0 & 3 & -3 & 0 & 3 \\ & 0 & 0 & \mp 3 & 0 \\ & & 0 & 3 & 0 \\ & & & 0 & 3 \\ & & & & 0 \end{pmatrix}, \quad (\text{A.7})$$

respectively. In fact, (A.6) and (A.7) are equivalent, as one can see by exchanging the second and the fourth rows and columns, i.e., by exchanging the first and the second U(1).

By applying the Theorem 2 to the first 4×4 part of (A.6), one can find that the plus sign in the double sign must be chosen. After all, the problem comes back to the case in (3.10). Then, we have to conclude again that there is no solution in the $U_c(3) \times U(1) \times U_L(2) \times U(1)^2$ model, even though the fermion embedding is generalized to (3.11) and (3.12).

A.3 Fluxes in the $U_c(3) \times U_L(2) \times U(1)^4$ model

We then study the $U_c(3) \times U_L(2) \times U(1)^4$ model considered in subsection 3.5. One can immediately find that the case (3.18) has no solution for q_l^{ab} , by applying the Theorem 1 to $q^{12} = -3$, $q^{23} = 3$, and $q^{13} = 3$.

For the case (3.17), by using the Theorem 1, the possible T^6 fluxes turn out to be as follows:

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 3 & 1 & 2 \\ & 0 & 3 & 0 & 0 & 0 \\ & & 0 & \pm 3 & x & 3-x \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}, \quad (\text{A.8})$$

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 3 & 3 & 0 \\ & 0 & 3 & 0 & 0 & 0 \\ & & 0 & u & x & y \\ & & & 0 & 0 & z \\ & & & & 0 & v \\ & & & & & 0 \end{pmatrix}, \quad (\text{A.9})$$

where x , y , and z can take an integer 0, 1, 2, or 3, while u and v can take 0, ± 1 , ± 2 , or ± 3 . In (A.9), $x + y + z = 3$ and $|u| + |v| = 3$ must be satisfied.

For (A.8), by applying the Theorem 2 to the first 4×4 part, i.e., q^{ab} with $1 \leq a, b \leq 4$, one can see that the plus sign in the double sign must be chosen. Then, q^{ab} with $1 \leq a, b \leq 4$ are the same as (3.6), as one can see by exchanging the second and the third rows and columns in q^{ab} . Thus, we can construct solutions for (A.8) by using the solutions in Table 2. There is one solution. The T^2 fluxes are written in the first row in Table 15. The corresponding T^6 fluxes

are (A.8) with $x = 1$, i.e.,

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 3 & 1 & 2 \\ & 0 & 3 & 0 & 0 & 0 \\ & & 0 & 3 & 1 & 2 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}. \quad (\text{A.10})$$

For (A.9), by applying the Theorem 2 to the first 4×4 part, i.e., q^{ab} with $1 \leq a, b \leq 4$, one can find that u must take -1 or 3 . In the same way, x must be -1 or 3 . As we showed below (3.10), there is no solution for $(u, x) = (3, 3)$. Since x must be positive, the remaining possibility is $(u, x) = (-1, 3)$. It then follows from $x + y + z = 3$ and $|u| + |v| = 3$ that $y = z = 0$ and $|v| = 2$. One can construct solutions by using those in Table 2 and Table 14. There is one solution. The T^2 fluxes are written in the second row in Table 15. The corresponding T^6 fluxes are

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 3 & 3 & 0 \\ & 0 & 3 & 0 & 0 & 0 \\ & & 0 & -1 & 3 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & -2 \\ & & & & & 0 \end{pmatrix}. \quad (\text{A.11})$$

For the case (3.16), by using the Theorem 1, the possible T^6 fluxes turn out to be as follows:

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 1 & 2 & 3 \\ & 0 & 3 & 0 & 0 & 0 \\ & & 0 & u & v & 3 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}, \quad (\text{A.12})$$

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 0 & 3 & 3 \\ & 0 & 1 & 2 & 0 & 0 \\ & & 0 & 0 & u & x \\ & & & 0 & v & 3-x \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix}, \quad (\text{A.13})$$

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}
$\begin{pmatrix} -1 & -2 & -1 & -1 & -1 \\ & -1 & 0 & 0 & 0 \\ & & 1 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & -2 & -1 & -1 & -1 \\ & -1 & 0 & 0 & 0 \\ & & 1 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} -3 & 0 & 3 & 1 & 2 \\ & 3 & 6 & 4 & 5 \\ & & 3 & 1 & 2 \\ & & & -2 & -1 \\ & & & & 1 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 & -1 & 1 & -1 \\ & 1 & 0 & 2 & 0 \\ & & -1 & 1 & -1 \\ & & & 2 & 0 \\ & & & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & -2 & -1 & 1 & 0 \\ & -3 & -2 & 0 & -1 \\ & & 1 & 3 & 2 \\ & & & 2 & 1 \\ & & & & -1 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 3 & 3 & 2 \\ & -1 & 0 & 0 & -1 \\ & & 1 & 1 & 0 \\ & & & 0 & -1 \\ & & & & -1 \end{pmatrix}$

Table 15: Fluxes in each T^2 in the $U_c(3) \times U_L(2) \times U(1)^4$ model with the fermion embedding (3.17). The first and the second rows of this table correspond to the T^6 fluxes (A.10) and (A.11), respectively. The diagonal elements are omitted.

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 3 & 0 & 3 \\ & 0 & 3 & 0 & 0 & 0 \\ & & 0 & u & v & x \\ & & & 0 & w & 0 \\ & & & & 0 & 3-x \\ & & & & & 0 \end{pmatrix}, \quad (\text{A.14})$$

where x can take an integer 0, 1, 2, or 3, while u , v , and w can take 0, ± 1 , ± 2 , or ± 3 . In (A.12) and (A.13), $|u| + |v| = 3$ is required. In (A.14), $|u| + |v| + |w| = 3$ is required.

Since (A.12) has a similar structure to (A.8), as can be seen by permuting the last three columns, one can find solutions in the same way. There are three solutions: The T^2 fluxes of the first row in Table 16 yield the T^6 fluxes (A.15). The second and the third rows in Table 16 give (A.16).

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 1 & 2 & 3 \\ & 0 & 3 & 0 & 0 & 0 \\ & & 0 & 1 & 2 & 3 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \quad (\text{A.15})$$

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 1 & 2 & 3 \\ & 0 & 3 & 0 & 0 & 0 \\ & & 0 & -3 & 0 & 3 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \quad (\text{A.16})$$

For (A.13), q^{ab} with $a, b = 1, 2, 5, 6$ are the same as (3.8), and thus one can use the results in Table 3. We just simply find solutions for the first 4×4 part in (A.13). We then combine the results, and obtain solutions for (A.13). There are three solutions: The T^2 fluxes of the fourth and the fifth rows in Table 16 yield T^6 fluxes (A.17). The sixth row in Table 16 gives (A.18).

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 0 & 3 & 3 \\ & 0 & 1 & 2 & 0 & 0 \\ & & 0 & 0 & -3 & 1 \\ & & & 0 & 0 & 2 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \quad (\text{A.17})$$

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 0 & 3 & 3 \\ & 0 & 1 & 2 & 0 & 0 \\ & & 0 & 0 & 1 & 1 \\ & & & 0 & 2 & 2 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \quad (\text{A.18})$$

For (A.14), q^{ab} with $a, b = 1, 2, 3, 4, 6$ are the same as the first 5×5 part in (A.9). One can thus follow the same arguments there. There are nine solutions: The T^2 fluxes in Table 17 yield the T^6 fluxes (A.19), and those in Table 18 give (A.20).

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 3 & 0 & 3 \\ & 0 & 3 & 0 & 0 & 0 \\ & & 0 & -1 & \pm 2 & 3 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \quad (\text{A.19})$$

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 3 & 0 & 3 \\ & 0 & 3 & 0 & 0 & 0 \\ & & 0 & -1 & 0 & 3 \\ & & & 0 & \pm 2 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \quad (\text{A.20})$$

In all of the above solutions (A.10), (A.11), (A.15)-(A.20), some of the fermion species are placed in several matrix elements, i.e., not all of the fermion species can be put in a single place with three generations. This is inevitable since there is no solution for (3.10). It may give some interesting results in the flavor structure.

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\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}
$\begin{pmatrix} -1 & -2 & -1 & -1 & -1 \\ & -1 & 0 & 0 & 0 \\ & & 1 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & -2 & -1 & -1 & -1 \\ & -1 & 0 & 0 & 0 \\ & & 1 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} -3 & 0 & 1 & 2 & 3 \\ & 3 & 4 & 5 & 6 \\ & & 1 & 2 & 3 \\ & & & 1 & 2 \\ & & & & 1 \end{pmatrix}$
$\begin{pmatrix} -1 & -2 & -1 & -1 & -1 \\ & -1 & 0 & 0 & 0 \\ & & 1 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & -2 & 1 & -2 & -1 \\ & -1 & 2 & -1 & 0 \\ & & 3 & 0 & 1 \\ & & & -3 & -2 \\ & & & & 1 \end{pmatrix}$	$\begin{pmatrix} -3 & 0 & -1 & 1 & 3 \\ & 3 & 2 & 4 & 6 \\ & & -1 & 1 & 3 \\ & & & 2 & 4 \\ & & & & 2 \end{pmatrix}$
$\begin{pmatrix} 1 & 0 & -1 & -1 & -1 \\ & -1 & -2 & -2 & -2 \\ & & -1 & -1 & -1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 1 & 2 & 3 \\ & -1 & -2 & -1 & 0 \\ & & -1 & 0 & 1 \\ & & & 1 & 2 \\ & & & & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 2 & -1 & -1 & -1 \\ & 3 & 0 & 0 & 0 \\ & & -3 & -3 & -3 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 & 0 & -1 & 1 \\ & 1 & 1 & 0 & 2 \\ & & 0 & -1 & 1 \\ & & & -1 & 1 \\ & & & & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & -1 & -1 & 1 \\ & 1 & -2 & -2 & 0 \\ & & -3 & -3 & -1 \\ & & & 0 & 2 \\ & & & & 2 \end{pmatrix}$	$\begin{pmatrix} 3 & 4 & 2 & 3 & 3 \\ & 1 & -1 & 0 & 0 \\ & & -2 & -1 & -1 \\ & & & 1 & 1 \\ & & & & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 & 0 & 3 & 1 \\ & 1 & 1 & 4 & 2 \\ & & 0 & 3 & 1 \\ & & & 3 & 1 \\ & & & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ & -1 & -1 & 0 & 0 \\ & & 0 & 1 & 1 \\ & & & 1 & 1 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 1 & 1 & 3 \\ & -1 & -2 & -2 & 0 \\ & & -1 & -1 & 1 \\ & & & 0 & 2 \\ & & & & 2 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 & 0 & 1 & -1 \\ & 1 & 1 & 2 & 0 \\ & & 0 & 1 & -1 \\ & & & 1 & -1 \\ & & & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 1 & -1 \\ & -1 & -1 & 0 & -2 \\ & & 0 & 1 & -1 \\ & & & 1 & -1 \\ & & & & -2 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 1 & 3 & 3 \\ & -1 & -2 & 0 & 0 \\ & & -1 & 1 & 1 \\ & & & 2 & 2 \\ & & & & 0 \end{pmatrix}$

Table 16: Fluxes in each T^2 in the $U_c(3) \times U_L(2) \times U(1)^4$ model with the fermion embedding (3.16) (Part 1). The first row of this table corresponds to the T^6 fluxes (A.15), the second and the third rows to (A.16), the fourth and the fifth rows to (A.17), and the sixth row to (A.18). The diagonal elements are omitted.

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}
$\begin{pmatrix} -1 & -2 & -1 & 0 & -1 \\ & -1 & 0 & 1 & 0 \\ & & 1 & 2 & 1 \\ & & & 1 & 0 \\ & & & & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 & 1 & 1 \\ & 1 & 2 & 0 & 0 \\ & & 1 & -1 & -1 \\ & & & -2 & -2 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 & -1 & -1 & -3 \\ & -3 & -4 & -4 & -6 \\ & & -1 & -1 & -3 \\ & & & 0 & -2 \\ & & & & -2 \end{pmatrix}$
$\begin{pmatrix} -1 & -2 & -1 & -1 & -1 \\ & -1 & 0 & 0 & 0 \\ & & 1 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 & 0 & 1 \\ & 1 & 2 & -1 & 0 \\ & & 1 & -2 & -1 \\ & & & -3 & -2 \\ & & & & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 & -1 & -1 & -3 \\ & -3 & -4 & -4 & -6 \\ & & -1 & -1 & -3 \\ & & & 0 & -2 \\ & & & & -2 \end{pmatrix}$
$\begin{pmatrix} -1 & -2 & -1 & -1 & -1 \\ & -1 & 0 & 0 & 0 \\ & & 1 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 & 0 & 1 \\ & 1 & 2 & -1 & 0 \\ & & 1 & -2 & -1 \\ & & & -3 & -2 \\ & & & & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 & -1 & 1 & -3 \\ & -3 & -4 & -2 & -6 \\ & & -1 & 1 & -3 \\ & & & 2 & -2 \\ & & & & -4 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 & -1 & 1 & 1 \\ & 1 & 0 & 2 & 2 \\ & & -1 & 1 & 1 \\ & & & 2 & 2 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -2 & -1 & 0 & 1 \\ & -3 & -2 & -1 & 0 \\ & & 1 & 2 & 3 \\ & & & 1 & 2 \\ & & & & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 3 & 3 & 3 \\ & -1 & 0 & 0 & 0 \\ & & 1 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 & -1 & -1 & 1 \\ & 1 & 0 & 0 & 2 \\ & & -1 & -1 & 1 \\ & & & 0 & 2 \\ & & & & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & -2 & -1 & 0 & 1 \\ & -3 & -2 & -1 & 0 \\ & & 1 & 2 & 3 \\ & & & 1 & 2 \\ & & & & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 3 & 3 & 3 \\ & -1 & 0 & 0 & 0 \\ & & 1 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$

Table 17: Fluxes in each T^2 in the $U_c(3) \times U_L(2) \times U(1)^4$ model with the fermion embedding (3.16) (Part 2). The solutions in this table correspond to the T^6 fluxes (A.19). The diagonal elements are omitted.

\hat{q}_1^{ab}	\hat{q}_2^{ab}	\hat{q}_3^{ab}
$\begin{pmatrix} -1 & -2 & -1 & -2 & -1 \\ & -1 & 0 & -1 & 0 \\ & & 1 & 0 & 1 \\ & & & -1 & 0 \\ & & & & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 & 1 & 1 \\ & 1 & 2 & 0 & 0 \\ & & 1 & -1 & -1 \\ & & & -2 & -2 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 & -1 & 0 & -3 \\ & -3 & -4 & -3 & -6 \\ & & -1 & 0 & -3 \\ & & & 1 & -2 \\ & & & & -3 \end{pmatrix}$
$\begin{pmatrix} -1 & -2 & -1 & 0 & -1 \\ & -1 & 0 & 1 & 0 \\ & & 1 & 2 & 1 \\ & & & 1 & 0 \\ & & & & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 3 & 1 & 1 \\ & 1 & 2 & 0 & 0 \\ & & 1 & -1 & -1 \\ & & & -2 & -2 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 & -1 & 0 & -3 \\ & -3 & -4 & -3 & -6 \\ & & -1 & 0 & -3 \\ & & & 1 & -2 \\ & & & & -3 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 & -1 & 0 & 1 \\ & 1 & 0 & 1 & 2 \\ & & -1 & 0 & 1 \\ & & & 1 & 2 \\ & & & & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & -2 & -1 & 1 & 1 \\ & -3 & -2 & 0 & 0 \\ & & 1 & 3 & 3 \\ & & & 2 & 2 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 3 & 4 & 3 \\ & -1 & 0 & 1 & 0 \\ & & 1 & 2 & 1 \\ & & & 1 & 0 \\ & & & & -1 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 & -1 & 0 & 1 \\ & 1 & 0 & 1 & 2 \\ & & -1 & 0 & 1 \\ & & & 1 & 2 \\ & & & & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & -2 & -1 & 1 & 1 \\ & -3 & -2 & 0 & 0 \\ & & 1 & 3 & 3 \\ & & & 2 & 2 \\ & & & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 2 & 3 & 2 & 3 \\ & -1 & 0 & -1 & 0 \\ & & 1 & 0 & 1 \\ & & & -1 & 0 \\ & & & & 1 \end{pmatrix}$

Table 18: Fluxes in each T^2 in the $U_c(3) \times U_L(2) \times U(1)^4$ model with the fermion embedding (3.16) (Part 3). The solutions in this table correspond to the T^6 fluxes (A.20). The diagonal elements are omitted.

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